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**EFFECTS OF SPANWISE VARIATION  
OF GUST VELOCITY ON ALLEVIATION  
SYSTEM DESIGNED FOR UNIFORM  
GUST VELOCITY ACROSS SPAN**

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GUST VELOCITY ON ALLEVIATION SYSTEM DESIGNED FOR  
UNIFORM GUST VELOCITY ACROSS SPAN

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SUMMARY

An analytical study has been made of the effectiveness of a gust-alleviation system in turbulent air. The system consisted of a vane mounted ahead of the wing to sense vertical gust velocity and of flaps driven in response to the vane deflection to reduce the accelerations induced by the gusts. The primary purpose of the study was to examine the effectiveness in two-dimensional turbulence (spanwise variation of gust velocity) of a system which was designed on the basis of one-dimensional turbulence (no spanwise variation of gust velocity). The results of the study indicated that the performance, as measured by the percent reduction in the mean-square lift on the wing, is about the same whether the turbulence is one- or two-dimensional, when the system is designed on the basis of optimum alleviation in one-dimensional turbulence.

INTRODUCTION

When an airplane encounters atmospheric turbulence, the passengers are sometimes subjected to uncomfortable motions. Consequently, it is desirable to develop gust-alleviation systems to improve ride qualities in rough air. Although gusts are generally isotropic in nature, analyses of gust-alleviation systems are usually based on gust-velocity variations only in the direction of flight (that is, no spanwise variation). This assumption, made in order to simplify the analyses, is valid if the wing span is small compared with the scale of turbulence (ref. 1). However, it appears that the assumption would become less valid as the wing span increases relative to the scale of turbulence.

The purpose of the present paper is to make a brief analysis of the applicability of results obtained by assuming the gust velocity to be uniform across the span (one-dimensional turbulence) to cases in which the gust velocity varies across the span (two-dimensional turbulence). The analysis is based on a system consisting of a vane to sense gusts and of a wing with flaps. The flaps are deflected in such a manner as to counteract the increment in wing lift due to the gusts. This study compares the mean-square values of the difference between the lift on the wing due to the vertical gust

velocity and the lift produced by the flaps for the following conditions:

- (1) Wing in one-dimensional turbulence, with flap gearing selected for maximum gust-load alleviation
- (2) Wing in two-dimensional turbulence, with flap gearing selected on the basis of one-dimensional turbulence
- (3) Wing in two-dimensional turbulence, with flap gearing selected for maximum gust-load alleviation on the basis of two-dimensional turbulence

### SYMBOLS

b	wing span
$C_L$	lift coefficient
$C_{L_\alpha}$	lift-curve slope, $\frac{\partial C_L}{\partial \alpha}$
$C_{L_{\delta_f}}$	change in wing lift coefficient with flap deflection, $\frac{\partial C_L}{\partial \delta_f}$
e	lift on wing, $L_W - L_f$
$\overline{e^2}$	mean-square lift on wing
$g(\sigma)$	nondimensional correlation function of vertical gust velocity
$h(x)$	impulse-response function of flap system
k	nondimensional natural frequency of flap servo, $\frac{\omega_n L}{U}$
K	gain constant, $\frac{C_{L_{\delta_f}}}{C_{L_\alpha}} K_1$
$K_1$	gain constant in flap control system
L	scale of turbulence

$L_f$	lift on wing due to flap deflection
$L_w$	lift on wing due directly to vertical gust velocity
$q$	dynamic pressure
$r$	percentage reduction in mean-square lift on basic wing when alleviation system is used, $\frac{\psi_1(0) - \tilde{\psi}_e(0)}{\psi_1(0)} \times 100$
$S$	wing surface area
$t$	time
$U$	constant forward velocity of wing
$w$	vertical gust velocity
$\overline{w^2}$	mean-square vertical gust velocity
$X, Y$	reference axes
$x, y$	coordinates
$y^*$	nondimensional spanwise coordinate, $\frac{y}{b/2}$
$\alpha$	angle of attack
$\alpha_g$	angle of attack of wing due to vertical gust velocity
$\alpha_v$	angle of attack of vane due to vertical gust velocity
$\gamma$	lift weighting function
$\delta(x)$	Dirac delta function
$\delta_f$	flap deflection
$\zeta$	damping ratio in flap control system

$l$	vane location forward of aerodynamic center of wing
$l^*$	nondimensional vane location, $l/b$
$\lambda$	dummy variable of integration
$\sigma$	distance between two points in space or along flight direction
$\psi_e$	correlation function of $e(x)$
$\tilde{\psi}_e$	nondimensional correlation function of $e(x)$
$\psi_w$	correlation function of vertical gust velocity
$\Omega_n$	spatial natural frequency of flap servo in radians per unit of distance
$\omega_n$	natural frequency of flap servo

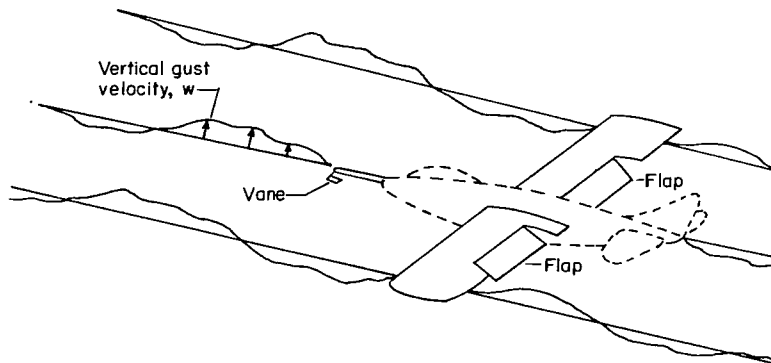
All primed values denote nondimensionalization by the scale of turbulence  $L$ ; for example,  $b' = \frac{b}{L}$ . The symbol  $\Delta$  preceding a quantity means an incremental value; for example,  $\Delta x = x_2 - x_1$ .

## STATEMENT OF PROBLEM

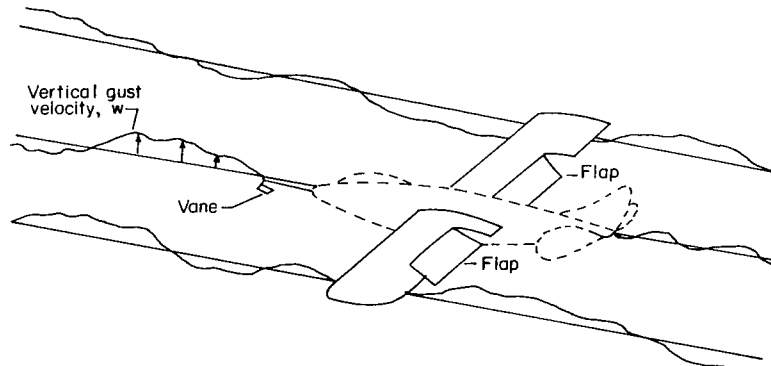
The alleviation system of this study consists of flaps on an airplane wing and a vane mounted on a boom ahead of the wing to sense the vertical gust velocity. The flaps are deflected either up or down to produce a lift opposite to that produced by the gust. In some cases, the flaps were assumed to deflect in phase with the vane; in others, the flaps were assumed to be operated by a linear second-order servomechanism.

Figure 1 shows the alleviation system being operated in two types of turbulence. In figure 1(a) the gust velocity varies randomly in the flight direction but is uniform across the span; that is, all velocity waves perpendicular to the span are identical (one-dimensional turbulence). In figure 1(b) the gust velocity varies randomly across the span as well as in the flight direction (two-dimensional turbulence).

Notice that the gust velocity hitting the vane is not necessarily equal to the gust velocities at various points along the wing. For this reason, the flap system cannot generally be expected to eliminate the lift due to the gust totally if it responds to the vane deflection. A measure of the residual lift on the wing can be estimated by using the statistical properties of gusts to calculate the statistical properties of the residual lift. (See refs. 2 and 3, for example.) The statistical property of interest in the present study is the mean square of the difference between the lift on the wing due to the vertical gust velocity and the lift produced by the flaps.



(a) One-dimensional turbulence (uniform gust velocity across wing span).



(b) Two-dimensional turbulence (spanwise variation of gust velocity).

Figure 1.- Airplane flying through turbulence.

## BASIC EQUATIONS

The coordinate system used in this study is shown in figure 2. A rigid wing is moving along the  $x$ -direction with constant forward velocity  $U$ . A vane is mounted a distance  $l$  ahead of the aerodynamic center to sense the gusts.

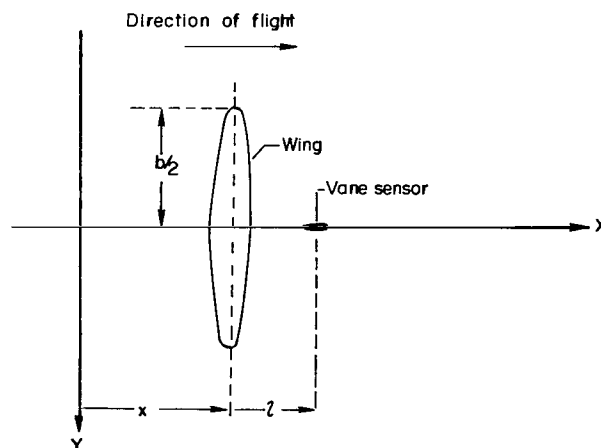


Figure 2.- Coordinate system.

## Lift on Wing-Flap System for Uniform Vertical

### Gust Velocity Across the Span

The lift increment of interest is the difference between the lift on the wing due to the vertical gust velocity and the lift produced by the flaps. Thus

$$e(x) = L_w(x) - L_f(x) = qSC_{L_\alpha} \alpha_g(x) - qSC_{L_{\delta_f}} \delta_f(x) \quad (1)$$

The dynamic response of the wing due to the gusts is not contained in this equation. Also, unsteady lift effects have been neglected.

The angle of attack caused by a vertical gust on the wing is given approximately by

$$\alpha_g(x) = \frac{w(x)}{U} \quad (2)$$

In this analysis it is assumed that the flap deflection is the response of a linear system, with the input variable proportional to the angle of attack of the sensing vane  $\alpha_v(x + l)$ . The flap deflection is given by the familiar Duhamel convolution integral as

$$\delta_f(x) = K_1 \int_0^\infty h(\lambda) \alpha_v(x + l - \lambda) d\lambda \quad (3)$$

where  $h(\lambda)$  is the flap response due to a unit impulsive vane angle of attack;  $h(\lambda) = 0$  for  $\lambda < 0$ . Subsequently, expressions are introduced for  $h(\lambda)$  which correspond to the following cases: (1) flaps moving in phase with the vane deflection and (2) flap deflection related to the vane deflection through a linear system of second order.

By assuming no lag in the response of the vane to a gust, the angle of attack of the vane can be expressed as

$$\alpha_v(x + l) = \frac{w(x + l)}{U} \quad (4)$$

Substituting equation (4) into equation (3) and then substituting equations (2) and (3) into equation (1) results in

$$e(x) = \frac{C_{L_\alpha} qS}{U} \left[ w(x) - K \int_0^\infty h(\lambda) w(x + l - \lambda) d\lambda \right] \quad (5)$$

where  $K = \frac{C_{L_{\delta_f}}}{C_{L_\alpha}} K_1$ .



Mean-Square Lift on the Wing-Flap System for Uniform  
Vertical Gust Velocity Across the Span

The mean-square lift for uniform vertical gust velocity across the span is obtained by averaging the square of the lift  $e(x)$  throughout the region of turbulence and is defined as

$$\psi_e(0) = \overline{e^2} = \lim_{x \rightarrow \infty} \frac{1}{2x} \int_{-x}^x [e(x)]^2 dx \quad (6)$$

This important statistic depends somewhat on the assumed model of atmospheric turbulence. In this study it is assumed that the correlation function between two vertical gust velocities is only a function of the separation between the two velocities along the flight direction. The form assumed for the correlation function is

$$\psi_w(\sigma) = \lim_{x \rightarrow \infty} \frac{1}{2x} \int_{-x}^x w(x) w(x + \sigma) dx = \overline{w^2} g(\sigma) \quad (7)$$

where  $\overline{w^2}$  is the mean-square vertical gust velocity and  $g(\sigma)$  is a nondimensional correlation function for vertical gusts which is a scalar function of the distance between the gust velocities along the flight direction.

Substituting equation (5) into equation (6) and making use of equation (7) results in the following expression for the nondimensional mean-square lift:

$$\tilde{\psi}_e(0) = \psi_1(0) - K\psi_2(0) - K\psi_3(0) + K^2\psi_4(0) \quad (8)$$

where

$$\tilde{\psi}_e(0) = \frac{\psi_e(0)}{\left(\frac{C_{L\alpha} q S}{U}\right)^2 \overline{w^2}} \quad (9)$$

$$\psi_1(0) = 1 \quad (10)$$

$$\psi_2(0) = \int_0^\infty h(\lambda) g(|l - \lambda|) d\lambda \quad (11)$$

$$\psi_3(0) = \int_0^\infty h(\lambda) g(|-l + \lambda|) d\lambda \quad (12)$$

and

$$\psi_4(0) = \int_0^\infty \int_0^\infty h(\lambda_1)h(\lambda_2)g(|\lambda_1 - \lambda_2|)d\lambda_1 d\lambda_2 \quad (13)$$

Note that  $\psi_2(0) = \psi_3(0)$ .

When  $K = 0$  in equation (8),  $\tilde{\psi}_e(0)$  represents the nondimensional mean-square lift on the wing with no alleviation ( $\psi_1(0)$ ). The remaining terms ( $K\psi_2(0)$ ,  $K\psi_3(0)$ , and  $K\psi_4(0)$ ) are associated with alleviation obtained by flap deflection. Since the purpose of the gust-alleviation system is to make  $e(x)$  as small as possible, it is desirable to choose  $K$  to minimize  $\tilde{\psi}_e(0)$ . This optimum value of  $K$  is obtained by differentiating equation (8) with respect to  $K$ , setting the result equal to zero, and solving for  $K$ . The result is

$$K = \frac{\psi_2(0) + \psi_3(0)}{2\psi_4(0)} = \frac{\psi_2(0)}{\psi_4(0)} \quad (14)$$

## APPLICATION OF EQUATIONS

### Gust Correlation Function

In order to compute the nondimensional mean-square lift  $\tilde{\psi}_e(0)$ , an expression must be assumed for the function  $g(\sigma)$ . For two vertical gust velocities located at points  $(x_1, y_1)$  and  $(x_2, y_2)$  on the wing, the function  $g(\sigma)$  is assumed to take the form

$$g(\sigma) = \exp\left(-2\frac{\sigma}{L}\right) \quad (15)$$

where

$$\sigma = |x_2 - x_1| \quad (16)$$

This correlation function was obtained from reference 3 primarily because of its simplicity. Results for other exponential forms of  $g(\sigma)$  can be obtained from the results of this study by proper interpretation of the scale of turbulence  $L$ .

A similar correlation function was also used in reference 4 in studying two-dimensional isotropic turbulence, the only difference being that  $\sigma$  was the true distance between the two points and not just the separation along the flight direction.

## Flap Response

The flap response to an impulse  $h(\lambda)$  depends on the assumed characteristics of the flaps and on the relation of the drive system to the angle sensed by the gust-sensing vane. Equations for the mean-square lift are derived for two types of flap response: (1) instantaneous in-phase response and (2) flap response described by a second-order linear differential equation.

For the in-phase flap response,  $h(\lambda)$  becomes a Dirac delta function which is written as

$$h(\lambda) = \delta(\lambda) \quad (17)$$

In order to examine the effect of lag in the flap response, it is assumed that the flap deflection  $\delta_f(x)$  is related to the angle of attack of the gust sensor  $\alpha_v(x + l)$  by means of a linear second-order differential equation of the form

$$\frac{d^2}{dx^2} \delta_f(x) + 2\zeta\Omega_n \frac{d}{dx} \delta_f(x) + \Omega_n^2 \delta_f(x) = K_1 \Omega_n^2 \alpha_v(x + l) \quad (18)$$

where  $\zeta$  is the damping ratio,  $\Omega_n$  is the natural frequency in radians per unit of distance, and  $K_1$  is a gain or proportionality constant.

For a damping ratio  $\zeta < 1$ , the solution of equation (18) is given by equation (3), in which

$$h(\lambda) = \frac{\Omega_n}{\sqrt{1 - \zeta^2}} \exp(-\zeta\Omega_n\lambda) \sin\left(\Omega_n\sqrt{1 - \zeta^2}\lambda\right) \quad (19)$$

Since  $x = Ut$ ,  $\Omega_n$  can be replaced in equation (19) by the equivalent expression  $\Omega_n = \frac{\omega_n}{U}$ , where  $\omega_n$  is expressed in radians per second. With this substitution, equation (19) becomes

$$h(\lambda) = \frac{\omega_n}{U\sqrt{1 - \zeta^2}} \exp\left(-\zeta\frac{\omega_n}{U}\lambda\right) \sin\left(\frac{\omega_n}{U}\sqrt{1 - \zeta^2}\lambda\right) \quad (20)$$

**Mean-Square Lift on the Wing-Flap System for Uniform  
Vertical Gust Velocity Across the Span**

In-phase flap response.- By using the properties of the Dirac delta function and equation (15), it is possible to obtain closed-form solutions to equations (8) to (13) and thereby obtain the nondimensional mean-square lift when the flaps move in phase with the vane (eq. (8)). The solution is

$$\tilde{\psi}_e(0) = 1 - 2K \exp(-2|\lambda^* b'|) + K^2 \quad (21)$$

where the term  $\lambda^* b'$  is used for convenience in subsequent comparisons.

The general expression for  $K$  which minimizes the mean-square lift is given by equation (14). For the in-phase flap response this expression reduces to

$$K = \exp(-2|\lambda^* b'|) \quad (22)$$

From equation (21) it can be seen that the nondimensional mean-square lift has a value of unity for the basic wing with no alleviation ( $K = 0$ ). The percentage reduction in the mean-square lift on the basic wing when the optimum value of  $K$  is used is then equal to

$$r = \left[ \exp(-4|\lambda^* b'|) \right] \times 100 \quad (23)$$

Lag in flap response.- The nondimensional mean-square lift which results when the flaps are driven by the second-order servomechanism is obtained by using equations (15) and (20) in conjunction with equations (8) to (13). The result is

$$\tilde{\psi}_e(0) = 1 - 2K\psi_2(0) + K^2\psi_4(0) \quad (24)$$

where

$$\psi_2(0) = \frac{k}{\sqrt{1 - \xi^2}} \int_0^\infty \exp(-k\xi\lambda') \sin(k\sqrt{1 - \xi^2}\lambda') \exp(-2|\lambda' - \lambda'|) d\lambda' \quad (25)$$

and

$$\psi_4(0) = \left( \frac{k}{\sqrt{1 - \xi^2}} \right)^2 \int_0^\infty \int_0^\infty \exp[-k\xi(\lambda_1' + \lambda_2')] \sin(k\sqrt{1 - \xi^2}\lambda_1') \sin(k\sqrt{1 - \xi^2}\lambda_2') \exp(-2|\lambda_1' - \lambda_2'|) d\lambda_1' d\lambda_2' \quad (26)$$

The optimum value of  $K$  for minimizing  $\tilde{\psi}_e(0)$  is given by equation (14). The percentage reduction in the mean-square lift on the basic wing is given by

$$r = \frac{\psi_1(0) - \tilde{\psi}_e(0)}{\psi_1(0)} \times 100 = \left[ 2K\psi_2(0) - K^2\psi_4(0) \right] \times 100 \quad (27)$$

#### Mean-Square Lift on the Wing-Flap System for Spanwise Variation of the Vertical Gust Velocity

For two-dimensional isotropic turbulence the nondimensional mean-square lift on the alleviated wing as derived in reference 4 is

$$\tilde{\psi}_e(0) = \psi_1(0) - K\psi_2(0) - K\psi_3(0) + K^2\psi_4(0) \quad (28)$$

where

$$\psi_1(0) = \frac{1}{4} \int_{-1}^1 \int_{-1}^1 \gamma(y_1^*) \gamma(y_2^*) g\left(\left|\frac{b}{2} \Delta y^*\right|\right) dy_1^* dy_2^* \quad (29)$$

$$\psi_2(0) = \frac{1}{2} \int_{-1}^1 \int_0^\infty \gamma(y^*) h(\lambda) g\left[\sqrt{(l - \lambda)^2 + (b/2)^2 (y^*)^2}\right] d\lambda dy^* \quad (30)$$

$$\psi_3(0) = \frac{1}{2} \int_{-1}^1 \int_0^\infty \gamma(y^*) h(\lambda) g\left[\sqrt{(-l + \lambda)^2 + (b/2)^2 (y^*)^2}\right] d\lambda dy^* \quad (31)$$

and

$$\psi_4(0) = \int_0^\infty \int_0^\infty h(\lambda_1) h(\lambda_2) g(|\Delta\lambda|) d\lambda_1 d\lambda_2 \quad (32)$$

The function  $\gamma(y^*)$  is called the lift distribution function and gives the lift increment due to a vertical gust velocity located at station  $y^*$  on the wing. As in reference 4, an elliptical lift distribution function is assumed; that is,

$$\gamma(y^*) = \frac{4}{\pi} \sqrt{1 - (y^*)^2} \quad (33)$$

The gust correlation function  $g(\sigma)$  between two vertical gust velocities located at points  $(x_1, y_1)$  and  $(x_2, y_2)$  is assumed to be given by equation (15), where

$$\sigma = \sqrt{(\Delta x)^2 + (\Delta y)^2} \quad (34)$$

The impulse flap response  $h(\lambda)$  is given by either equation (17) or equation (20).

### Summary of Applications

Equations (15) to (34) were used to compute the following:

- (1) The mean-square incremental lift on the wing, with the assumption of one-dimensional turbulence and the use of a flap gear ratio (that is, gain constant  $K$ ) selected for maximum gust-load alleviation
- (2) The mean-square incremental lift on the wing, with the assumption of two-dimensional turbulence and the use of a flap gear ratio selected on the basis of maximum gust-load alleviation in one-dimensional turbulence
- (3) The mean-square incremental lift on the wing, with the assumption of two-dimensional turbulence and the use of a flap gear ratio selected on the basis of maximum gust-load alleviation in two-dimensional turbulence

### RESULTS AND DISCUSSION

For clarity in presenting the results, three distinct cases are identified in the following table:

Parameter	Case 1	Case 2	Case 3
w . . . .	One-dimensional turbulence (random in flight direction (eqs. (15) and (16)) but uniform across span)	Two-dimensional turbulence (spanwise variation, eqs. (15) and (34))	Two-dimensional turbulence (spanwise variation, eqs. (15) and (34))
K . . . .	Optimum value, eq. (14)	Case 1 value, eq. (14)	Optimum value from ref. 4

In case 1, the alleviation system is designed for, and operated in, uniform gust velocity across the span; in case 2, the alleviation system is designed for uniform gust velocity across the span but is operated in gust velocity which varies across the span; and in case 3, the alleviation system is designed for, and operated in, gust velocity which varies across the span.

The results of the study are presented first for in-phase flap response (the flaps moving in phase with the vane) and then for lag in flap response.

## In-Phase Flap Response

**Nondimensional mean-square lift.**— Figure 3 shows the nondimensional mean-square lift on the wing  $\tilde{\psi}_e(0)$  for case 1 (one-dimensional turbulence) as a function of the ratio of wing span to the scale of turbulence ( $b'$ ) and vane location in fractions of wing span ( $l^*$ ). The dashed line refers to the nondimensional mean-square lift on the basic wing (no alleviation), whereas the solid lines refer to the alleviation results. The values of  $K$  used to generate the curves of figure 3 were chosen to minimize the mean-square lift on the wing. These values of  $K$ , as determined by equation (22), are plotted in figure 4.

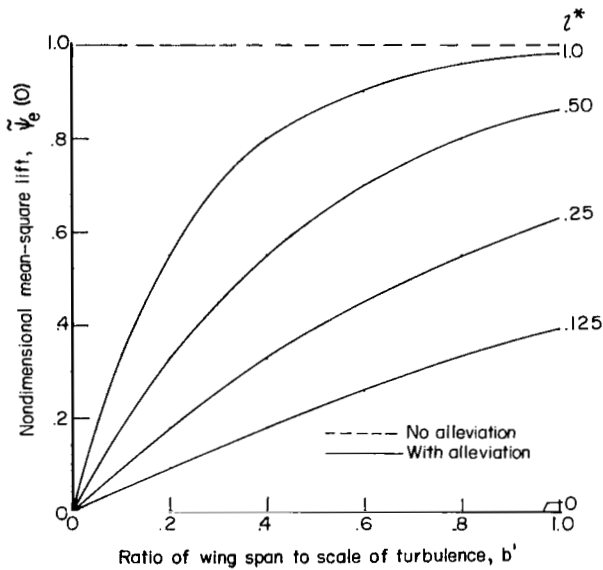


Figure 3.— Effect of alleviation system on the mean-square value of  $e(x)$  in case 1. (Flaps moving in phase with vane and optimum value of gain constant  $K$  used.)

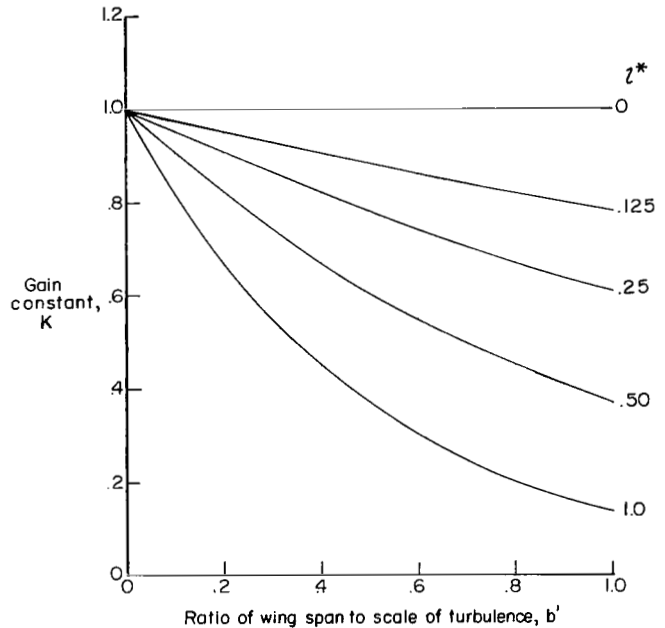


Figure 4.— Gain constant used to minimize the mean-square value of  $e(x)$  in case 1. (Flaps moving in phase with vane.)

Since the gust velocity is uniform across the span and the flaps respond immediately to the gust sensed by the vane, there is perfect alleviation for the condition  $l^* = 0$  and  $K = 1$ , as shown in figures 3 and 4. As the vane is moved farther ahead of the wing (as  $l^*$  increases), the alleviation system becomes less effective (fig. 3) since the flaps respond too soon. The optimum value of  $K$  becomes smaller as  $l^*$  and  $b'$  are increased (fig. 4).

Figure 5 shows the nondimensional mean-square lift  $\tilde{\psi}_e(0)$  for the wing flying through two-dimensional turbulence with and without the gust-alleviation system that was designed for maximum gust-load alleviation in one-dimensional turbulence. The

nondimensional mean-square lift is presented as a function of the ratio of wing span to the scale of turbulence  $b'$  and vane location in fractions of wing span  $l^*$ .

In figure 5 the dashed line refers to the nondimensional mean-square lift on the basic wing (no alleviation) and shows that as the wing span becomes larger relative to the scale of turbulence (as  $b'$  increases), the nondimensional mean-square lift decreases. This self-alleviation is due to a spanwise averaging effect; that is, the lift increments due to the various gust velocities tend to average out. The solid lines represent the results for the alleviation system with the flaps moving in phase with the vane. The values of  $K$  used to obtain these curves were the same as those in figure 4.

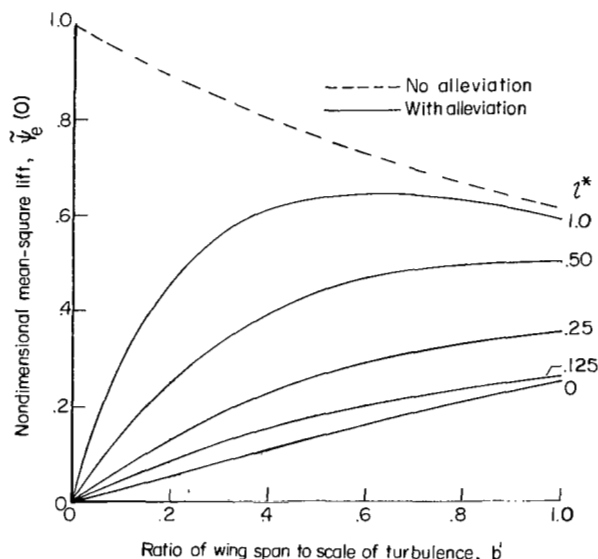


Figure 5.- Effect of alleviation system on the mean-square value of  $e(x)$  in case 2. (Flaps moving in phase with vane and gain constant  $K$  from case 1 used.)

By comparing the results of figures 5 and 3, it can be seen that the magnitude of the mean-square lift is about the same for very small values of  $b'$ . However, for large values of  $b'$  and  $l^*$ , there can be a considerable difference in the magnitude of  $\tilde{\psi}_e(0)$ . For example, let  $b' = 1$  and  $l^* = 1$ ; in figure 3,  $\tilde{\psi}_e(0) = 0.98$ , whereas in figure 5,  $\tilde{\psi}_e(0) = 0.59$ . The difference is about 40 percent.

Percentage reduction in mean-square lift.- The curves of figures 3 and 5 were used to determine the percent reduction in mean-square lift attainable for cases 1 and 2, respectively. The results shown in figure 6 indicate that, except for the condition  $l^* = 0$ , the predicted percent reduction in lift is almost the same for the two cases. In other words, if the gear ratio is selected on the basis of a one-dimensional-turbulence analysis, the anticipated percentage of gust alleviation in two-dimensional turbulence is about the



same as would be predicted for one-dimensional turbulence. This statement does not hold if  $l^*$  is very close to zero, since alleviation is perfect for uniform gust velocity across the span.

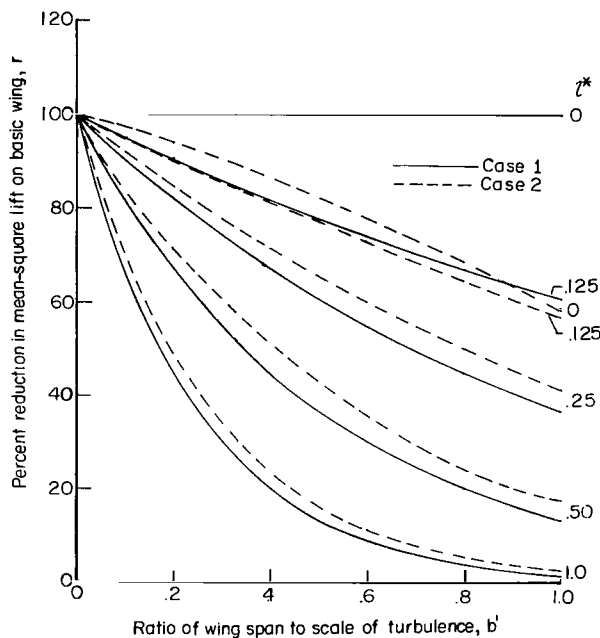


Figure 6.- Performance of alleviation system in case 1 and case 2. (Flaps moving in phase with vane.)

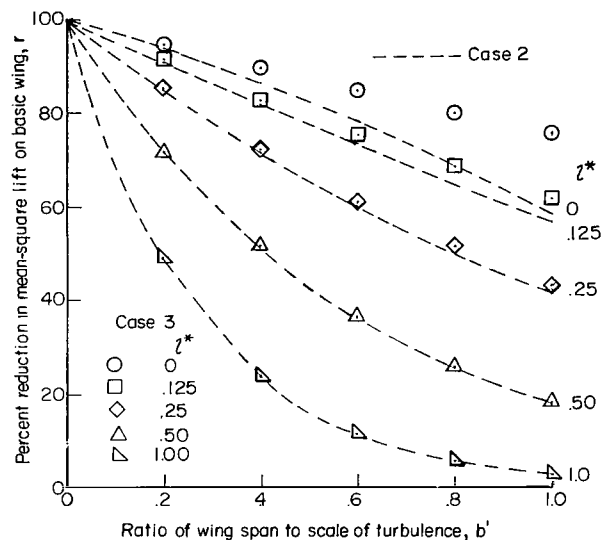


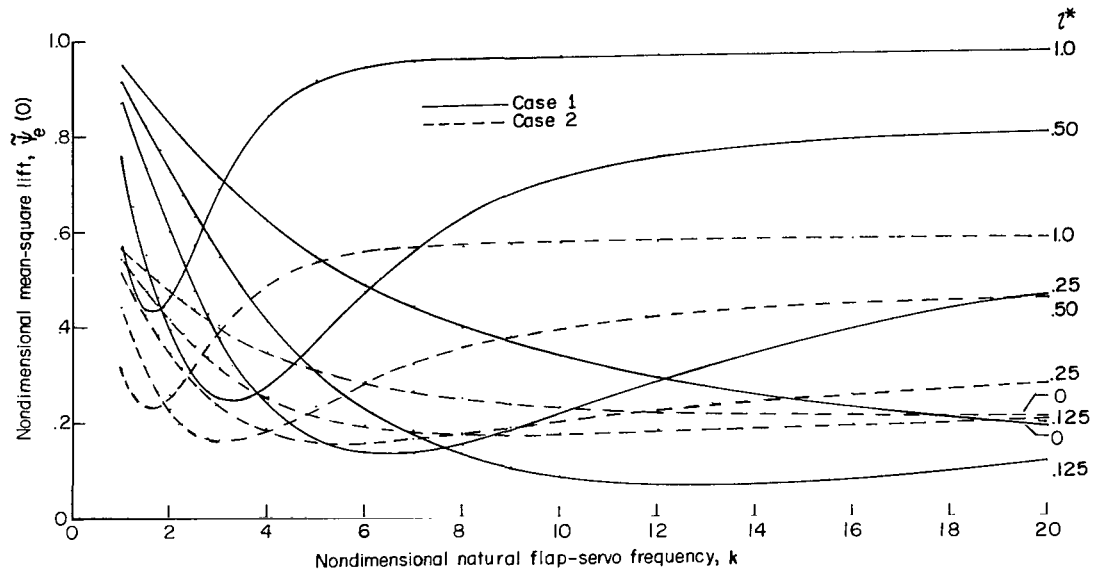
Figure 7.- Performance of alleviation system in case 2 and case 3. (Flaps moving in phase with vane.)

Next, it was of interest to determine if further improvement of gust alleviation in two-dimensional turbulence could be obtained if the gear ratio were selected for optimal lift reduction in two-dimensional turbulence (case 3). A comparison (for cases 2 and 3) is made in figure 7. The results indicate that the value of the gear ratio  $K$  selected on the basis of a one-dimensional-turbulence analysis results in about the same percentage reductions in gust load (mean-square lift) as when  $K$  is selected on the basis of a two-dimensional-turbulence analysis.

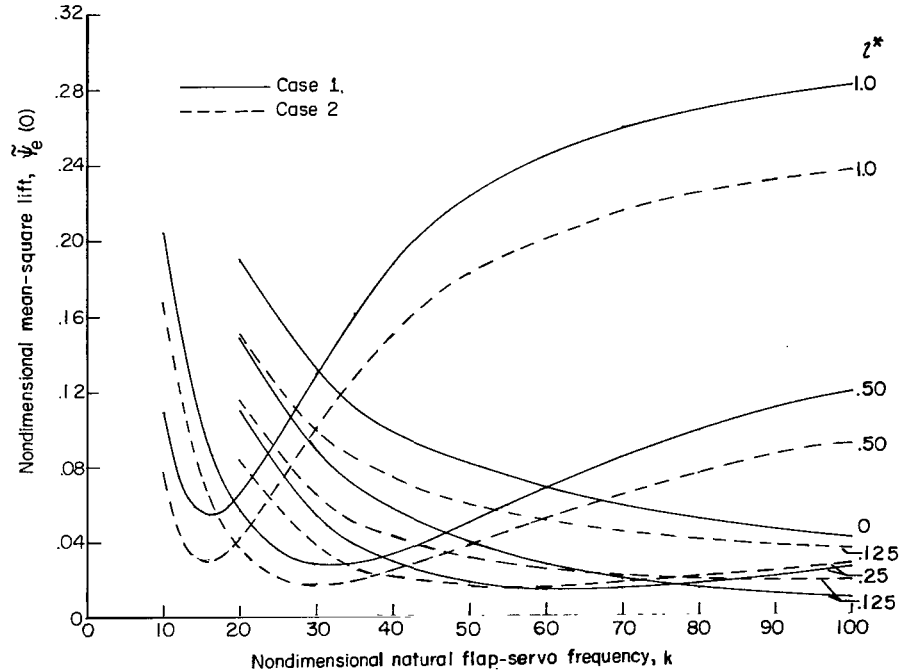
#### Effect of Lag in the Flap Response

Notice that as the vane is moved farther ahead of the wing, the gust alleviation shown in figure 3 becomes less. As noted previously, the reason is that the flaps move too soon. If the vane is mounted ahead of the wing, there should be a time delay in the flap response to allow the gust sensed by the vane to reach the wing. This lag can be incorporated into the system by varying the frequency of the servomechanism driving the flaps.

Nondimensional mean-square lift.- The value of  $\tilde{\psi}_e(0)$  in cases 1 and 2 is shown in figure 8 as a function of the nondimensional natural flap-servo frequency  $k$  and various vane locations. Results are presented for two values of nondimensional wing span;



(a)  $b' = 1.0$ .



(b)  $b' = 0.1$ .

Figure 8.- Effect of servo response of the alleviation system on the mean-square value of  $e(x)$  in case 1 and case 2.  $\zeta = 0.7$ .

namely,  $b' = 1.0$  and  $b' = 0.1$ . The values of  $K$  used to generate these curves were those values which minimize  $\tilde{\psi}_e(0)$  in case 1, as determined by equation (14). These values of  $K$  are plotted in figure 9.

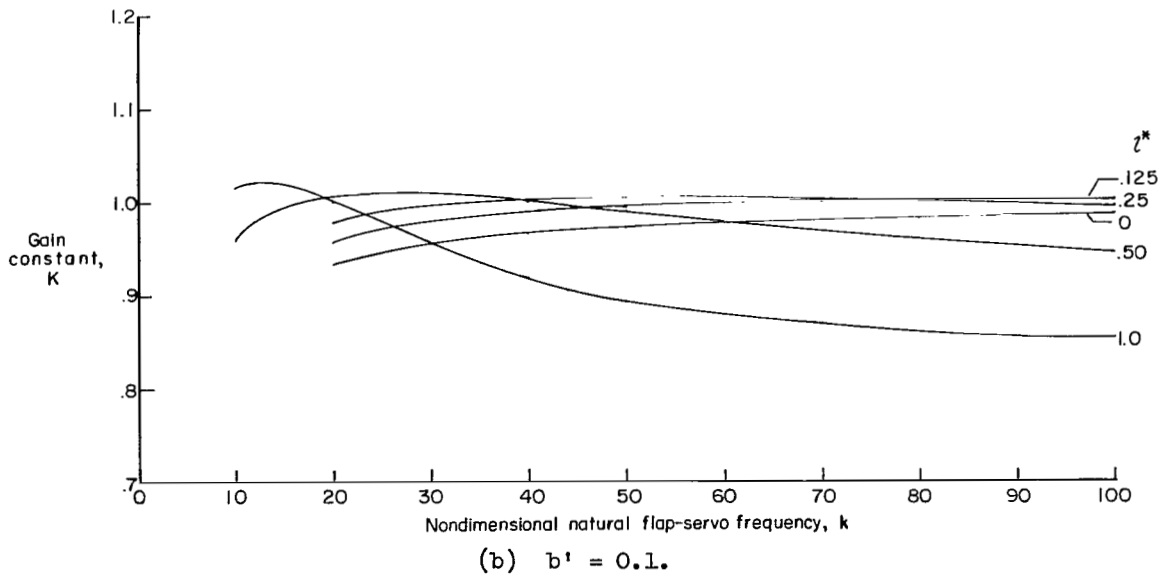
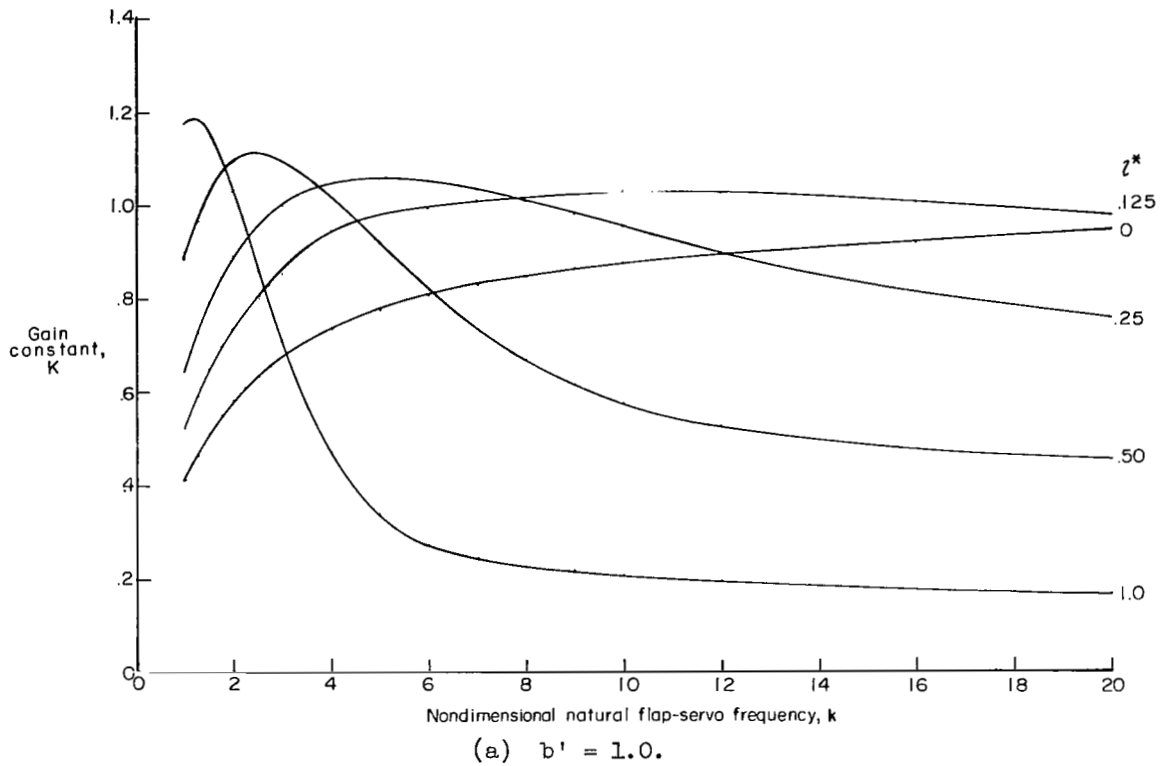


Figure 9.- Gain constant used to minimize the mean-square value of  $e(x)$  in case 1.  $\zeta = 0.7$ .

Figure 8(a) shows that if the wing span is on the same order of magnitude as the scale of turbulence ( $b' = 1$ ), the magnitude of the mean-square lift can differ appreciably in cases 1 and 2 for different servo frequencies and vane locations. On the other hand, figure 8(b) shows that if the wing span is small compared with the scale of turbulence ( $b' = 0.1$ ), the magnitude of the mean-square lift is about the same in each case.

Percentage reduction in mean-square lift.— Figure 10 shows the percentage reduction in the mean-square lift on the basic wing when the alleviation system is used for case 1 and case 2. The percentage reduction  $r$  is presented as a function of the non-dimensional flap-servo frequency  $k$  and vane location  $l^*$  for  $b' = 1.0$  and  $b' = 0.1$ . The solid curves refer to the reduction predicted for uniform gust velocity across the span (case 1), whereas the dashed curves refer to the reduction that results when the alleviation system of case 1 is used in two-dimensional turbulence (case 2).

Figure 10(a) shows the percentage reduction in mean-square lift when the wing span is on the same order of magnitude as the scale of turbulence ( $b' = 1$ ). Notice that, for vane locations  $l^* \geq 0.25$ , the percentage reduction in cases 1 and 2 is about the same at all frequencies of the flap servomechanism. As  $k$  becomes very small, there is good agreement for all the vane locations; as  $k$  becomes large, the values of  $r$  approach the in-phase results of figure 6, and the agreement becomes good for all vane locations  $l^* \geq 0.125$ .

Figure 10(b) shows that when the wing span is small compared with the scale of turbulence ( $b' = 0.1$ ), the percentage reduction in the mean-square lift is about the same in cases 1 and 2 for all vane locations and flap-servo frequencies. (Note the expanded scales, as compared with those in figure 10(a).)

The dashed lines of figure 10 (case 2) indicate the performance of the alleviation system in two-dimensional turbulence after it has been designed for one-dimensional turbulence. It is now of interest to determine how this performance varies when the alleviation system is both designed for and operated in two-dimensional turbulence (case 3).

The results of case 2 are compared with those of case 3 in figure 11 for  $b' = 1.0$  and  $b' = 0.1$ . The percentage reduction in the mean-square lift is presented as a function of the nondimensional flap-servo frequency for different vane locations. The results of case 2 are shown by the dashed lines, while those of case 3 are denoted by the various symbols.

The results in figure 11 show that, in general, the performance (as far as the percentage reduction in the mean-square lift is concerned) of the alleviation system in two-dimensional turbulence is about the same whether it is designed for one-dimensional turbulence or for two-dimensional turbulence.

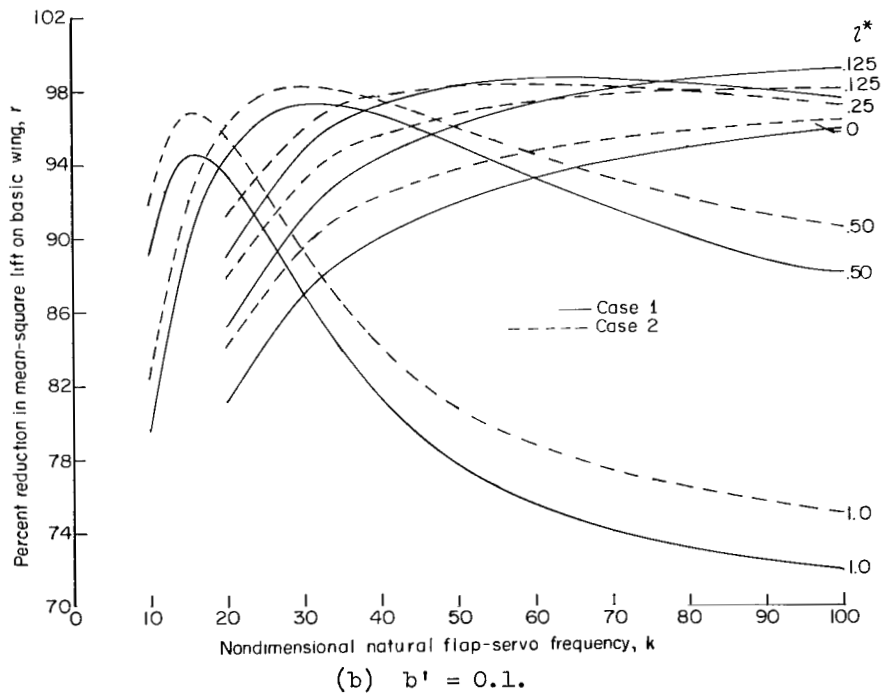
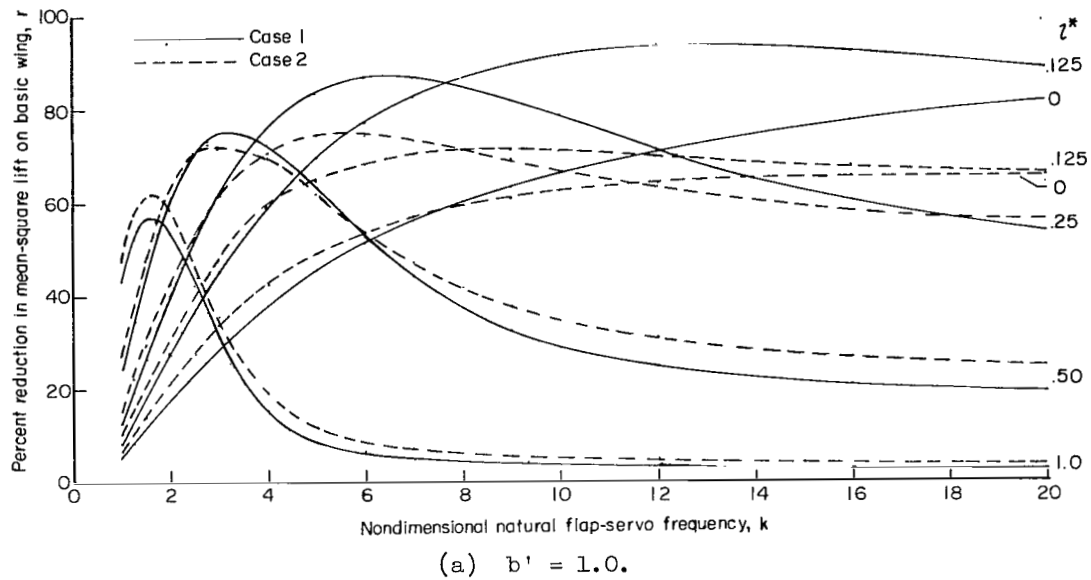
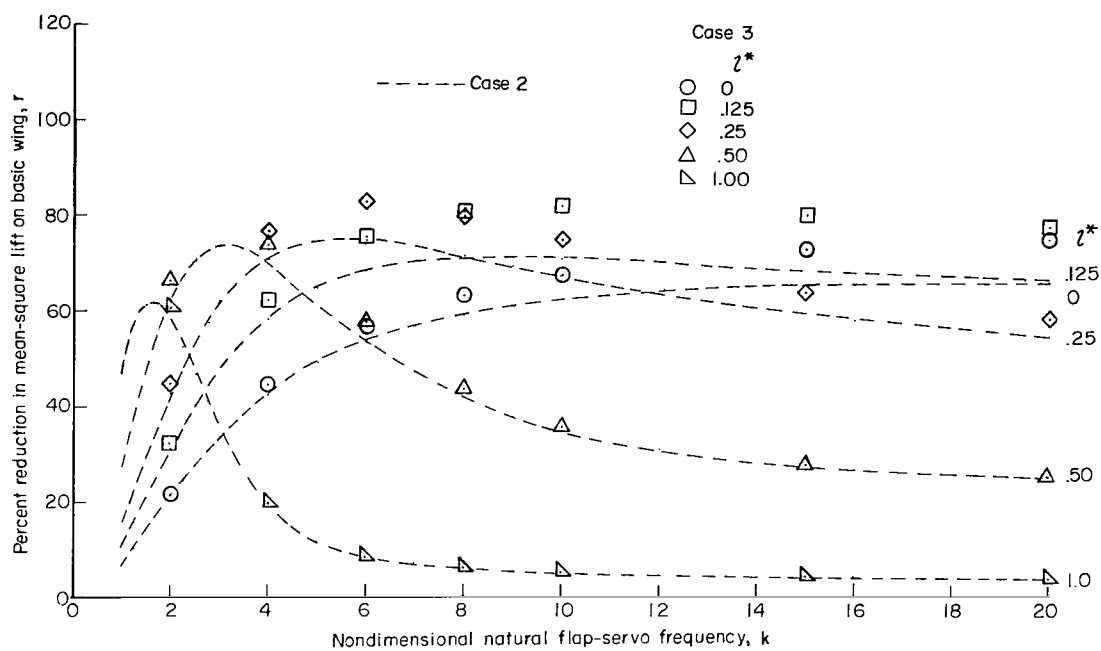
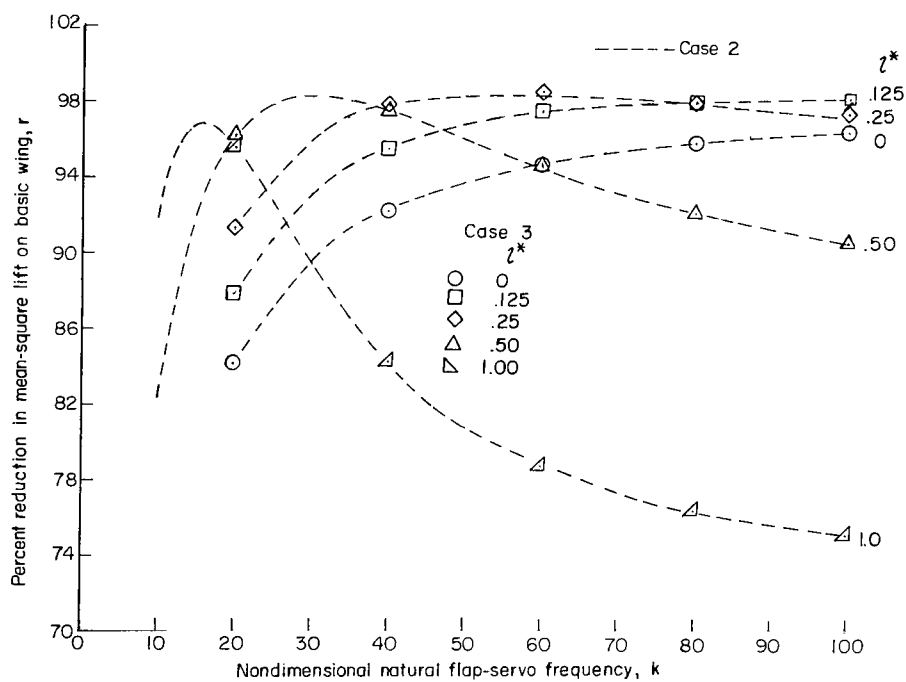


Figure 10.- Performance of alleviation system in case 1 and case 2.  $\zeta = 0.7$ .



(a)  $b' = 1.0$ .



(b)  $b' = 0.1$ .

Figure 11.- Performance of alleviation system in case 2 and case 3.  $\xi = 0.7$ .

The reduction in mean-square lift should improve passenger comfort, since reference 4 shows that most of this reduction occurs at the lower frequencies and reference 1 states that passenger comfort is primarily affected by the lower frequency gusts.

### CONCLUDING REMARKS

An analytical study has been made of a gust-alleviation system in which a vane is mounted ahead of an airplane wing to sense the vertical gust velocity. The flaps are then driven in response to a signal from the vane either instantaneously (in phase) or through a linear servomechanism to produce a lift opposite to that produced by the gust. The alleviation system was subjected to two types of turbulence: (1) the vertical gust velocity was assumed to vary randomly in the flight direction but to be uniform across the wing span (one-dimensional turbulence) and (2) the vertical gust velocity was allowed to vary randomly over the span as well as in the flight direction (two-dimensional turbulence).

The results of the study indicate that if the ratio of the wing span to the scale of turbulence is very small, the mean-square lift on the alleviated wing is approximately the same for one- and two-dimensional turbulence. However, as this ratio approaches unity, the magnitude of the mean-square lift can differ appreciably for the two types of turbulence.

It was also found that the performance of the alleviation system, as measured by the percentage reduction in the mean-square lift on the wing, was about the same for both gust models. Furthermore, an alleviation system designed for optimum performance in one-dimensional turbulence will also yield nearly optimum performance when subjected to two-dimensional turbulence.

Langley Research Center,  
National Aeronautics and Space Administration,  
Hampton, Va., April 21, 1971.

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